Solving Quadratics with the Quadratic Formula

① Make sure equation $= 0$ ② Find a, b, and c

③ Plug in values ④ Simplify discriminant ⑤ Finish simplifying

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$x^2 - 5x - 14 = 0$

Example **Solve $x^2 - 5x = 14$ by using the Quadratic Formula.**

① Rewrite the equation as -14 -14 ② $a = 1$ $b = -5$ $c = -14$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

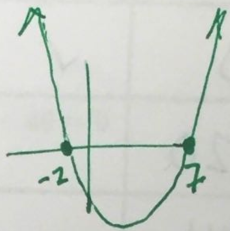
③ $= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ Replace a with 1, b with -5, and c with -14. $b^2 - 4ac = 81$

$= \frac{5 \pm \sqrt{81}}{2}$ Simplify.

$= \frac{5 \pm 9}{2}$ $+ \rightarrow \frac{5+9}{2} = \frac{14}{2} = 7$ $- \rightarrow \frac{5-9}{2} = \frac{-4}{2} = -2$

$=$ or

$\boxed{\text{The solutions are } -2 \text{ and } 7.}$



Roots and the Discriminant

Discriminant	The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the **discriminant**.

Roots of a Quadratic Equation

Discriminant	Type and Number of Roots
$b^2 - 4ac > 0$ perfect square (positive)	2 real solutions → rational
$b^2 - 4ac > 0$ NOT a perfect square	2 real solutions → irrational
$b^2 - 4ac = 0$	1 real solution
$b^2 - 4ac < 0$ (negative)	2 complex solutions (i)

$\sqrt{56}$

$\sqrt{4}\,\sqrt{14}$

$2\sqrt{14}$

Example **Find the value of the discriminant for each equation. Then describe the number and types of roots for the equation.**

a. $2x^2 + 5x + 3$
The discriminant is
$b^2 - 4ac = 5^2 - 4(2)(3)$ or 1.
The discriminant is a perfect square, so the equation has 2 rational roots.

b. $3x^2 - 2x + 5$
The discriminant is
$b^2 - 4ac = \underline{(-2)^2 - 4(3)(5) = -56}$
The discriminant is $\underline{\text{neg}}$, so the equation has $\underline{\text{2 complex solutions}}$

$x = \frac{-(-2) \pm \sqrt{-56}}{2(3)} = \frac{2 \pm i\sqrt{56}}{6} = \frac{2 \pm 2i\sqrt{14}}{6} = \boxed{\frac{1 \pm i\sqrt{14}}{3}}$